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Reentry Trajectory Optimization : Evolutionary Approach

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Abstract

This paper shows the application of Genetic Algorithm to Reentry trajectory optimization problem in the presence of constraints such as dynamic pressure and heat flux. The coupled non linear equations of motion of reentry body are difficult to solve using conventional methods like finding costate equations and then applying standard optimization methods. As an alternate, stochastic way of optimization through genetic algorithm is proposed. The results for a typical Reusable Launch Vehicle – Technology Demonstrator are in good match with that of Space Shuttle.

Nomenclature

C_D	=	drag coefficient
C_{Do}	=	constant
C_L	=	lift Coefficient
$C_{L\alpha}$	=	C_L as a function of α, /rad
D	=	drag force, N
F_T	=	thrust force, N
g_o	=	gravity at earth surface, m/s^2
g	=	gravity at some altitude, m/s^2
h	=	altitude, m
I_{sp}	=	specific impulse, N s/kg
k	=	constant
L	=	lift force, N
LH	=	local horizontal plane
M	=	total mass, kg
Q	=	dynamic pressure, Pa
q	=	heat flux, W/cm^2
r_o	=	radius of earth, m
r	=	radial vector, m
R_N	=	nose radius, m
S	=	area, m^2
U	=	control vector
V	=	velocity, m/s
α	=	angle of attack, deg
β	=	constant
ϕ	=	latitude, deg
γ	=	flight path angle, deg
ρ_o	=	density at earth surface, kg/m^3
ρ	=	density at some altitude,kg/m^3
θ	=	longitude, deg
σ	=	bank angle, deg
ψ	=	heading angle, deg

Introduction

The concept of Reusable Launch Vehicle (RLV) is evolved in order to minimize cost per unit payload. Nearly 70% of cost of a launch vehicle lies in structure and avionics system, recovering the same and using it for further flights (after refurbishing) will be less expensive. The technologies like TPS material, structure and avionics system for a full fledged RLV are highly challenging. As a cost effective measure, a smaller version of RLV is proposed called as Technology Demonstrator (RLV-TD) as given in Fig.1 which will validate the above mentioned technologies in a step by step manner.

Fig.1 RLV – Technology Demonstrator

In the reentry trajectory design one has to ensure that with minimum fuel consumption the desired landing site is reached satisfying constraints such as dynamic pressure and heat flux. Traditionally reentry trajectory optimization is carried out by non dimensionalising the equations of motion[1] and then solving it through gradient method using algorithms such as SGRA[2] , Brysons[3]. As the equations of motion of a reentry body are coupled non linear equation, it gives complicated co state equations which are difficult to solve and simulate. As an alternate, evolutionary approach is proposed for this type of problems.

Genetic Algorithms(GA)[4] are search algorithms based on the mechanism of natural selection. They lay on one of the most important principles of Darwin : survival of the fittest. Globally we make use of the population, submitted to many transformations. After some generations, the population endues no more; the best individual represents the optimal solution. GAs do not require

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derivative or continuity of a function to be evaluated. They are simple, fast converging and easy to implement and simulate.

Equations of motion of a reentry body

The equations of motion for a lifting trajectory are given[1,5] by (Fig.2)

$$\dot{V} = -g \sin\gamma - \frac{D}{M} - \frac{F_T}{M}$$
$$\dot{\gamma} = \left(\frac{V}{r} - \frac{g}{V}\right)\cos\gamma + \frac{L}{MV}\cos\sigma$$
$$\dot{\psi} = -\frac{V}{r}\cos\gamma\cos\psi\tan\phi + \frac{L}{MV}\frac{\sin\sigma}{\cos\gamma}$$
$$\dot{r} = V\sin\gamma$$
$$\dot{\theta} = \frac{V}{r}\frac{\cos\gamma\cos\psi}{\cos\phi}$$
$$\dot{\phi} = \frac{V}{r}\cos\gamma\sin\psi$$

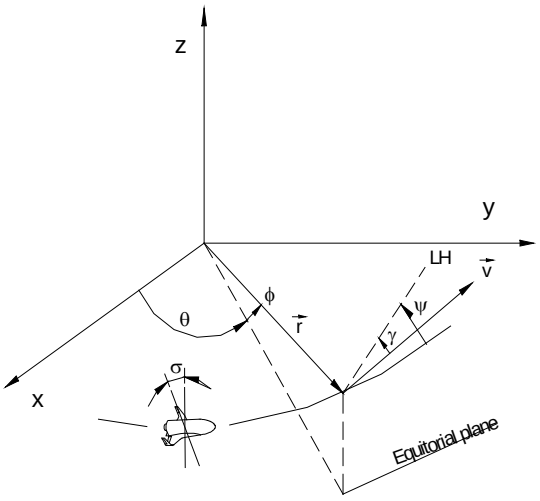

Fig.2 Definition of various parameters

The control vector is U=[α,σ]. The drag, the lift, the density and the gravity acceleration are assumed to have standard form :

$$D = \frac{1}{2}\rho V^2 C_D S$$
$$L = \frac{1}{2}\rho V^2 C_L S$$
$$\rho = \rho_o e^{-\beta h}$$
$$g = g_o\left(\frac{r_o}{r}\right)^2$$
$$r = r_o + h$$

The density is assumed to be zero above 120km altitude. The heat flux is given by

$$q = \frac{18300}{\sqrt{R_N}}(\rho)^{0.5}\left(\frac{V}{10^4}\right)^{3.05}$$

Genetic Algorithm

They are different from more normal optimization and search procedures in following ways[6,7]

- GAs work with coding of the parameter set, not the parameter themselves
- GAs search from a population of points, not a single point. They climb many peak in parallel and not go point by point.
- GAs use objective function information, not derivatives
- GAs use probabilistic transition rules, not deterministic rules.

Traditionally in GA, the number of variables in the objective function are coded into binary strings (0 and 1). Each bit is a gene and the complete string is called a chromosome. An initial population (of chromosome) is chosen randomly. Each chromosome represents a potential solution to the problem. At each generation, the chromosome of the population are evaluated from their fitness value and reproduced further using mutation and crossover.

In the present study, each chromosome vector is represented with floating point numbers. It has been shown that[6] floating point representation is faster, more consistent from run to run and provided a higher precision. Its performance can be enhanced by special operators (like boundary mutation) to achieve high accuracy.

Problem Statement

Minimize, the control force

$$J = \int U^2(t)\, dt$$

and meet constraints on Q and q.

The integration of equations of motion is carried out at 1 s interval upto 300 s. The problem is discretized into 300 steps $[t_1,t_2,..t_{300}]$ where we have to determine control vector U[α,σ] so as to satisfy constraints. The initial values of state vectors are specified. In the present study, cross range optimization is not carried out and as such σ = 0 is assumed. Also all the propellant is utilized from t=0 to t=200s.

Input Data

Mass at orbit = 1750 kg
Mass of propellant = 300 kg
Isp = 2696.8 N s/kg, S = 0.785 m^2, R_N = 0.25m
ρ_o = 1.23 kg/m^3, β = 1/7320, r_o = 6378000m
g_o = 9.80665 m/s^2
Initial [h,V,γ,θ,ϕ,ψ] is [150000,1500,0,0,0,0]
The aerodynamic model is assumed to be
$C_D = C_{Do} + kC_L^2$
C_{Do} = 0.4, k = 0.18
$C_L = C_{L\alpha} . \alpha$
$C_{L\alpha}$ = 7.0 /rad

Results and Discussion

The constraint boundaries for RLV-TD is given in Fig.3. The boundary where lift force is balancing gravity minus centrifugal force is called Equilibrium Glide Boundary. The vehicle will be following heat flux boundary (q) till about 25 km thereafter it will go along dynamic pressure (Q) boundary .

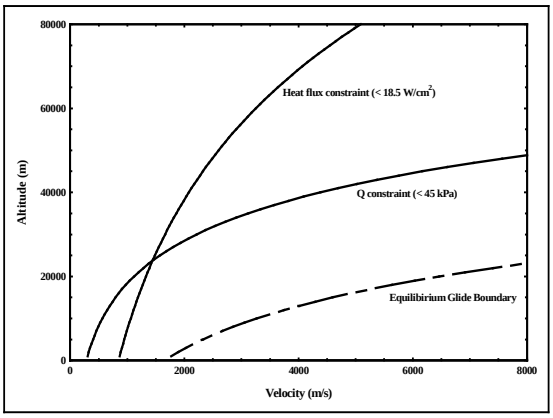

Fig.3 Boundary constraints

The unconstrained Q is > 300 kPa for RLV-TD. Using α as controlling parameter, Q can be controlled to Q < 50 kPa and Q < 45 kPa as given in Fig.4. The control angle history is given in Fig.5 for the two cases.

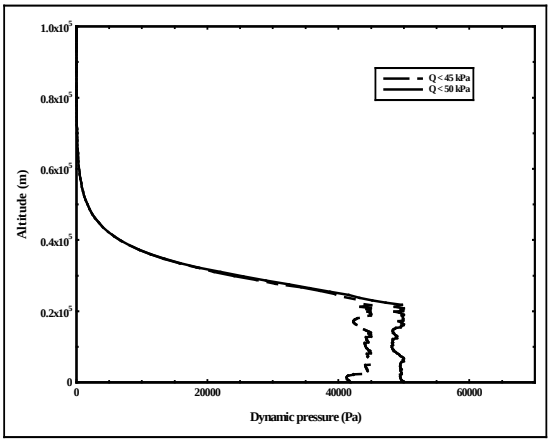

Fig.4 Q constraint only

In the next set of optimization two set of constraints were specified

(1) Q < 45 kPa

(2) q < 18.5 W/cm^2

The altitude, velocity, flight path angle and heat flux plot are given in Fig.6, Fig.7, Fig.8 and Fig.9 respectively. The control angle history is given in Fig.10. It is touching maximum permissible value of 30deg at 170 s.

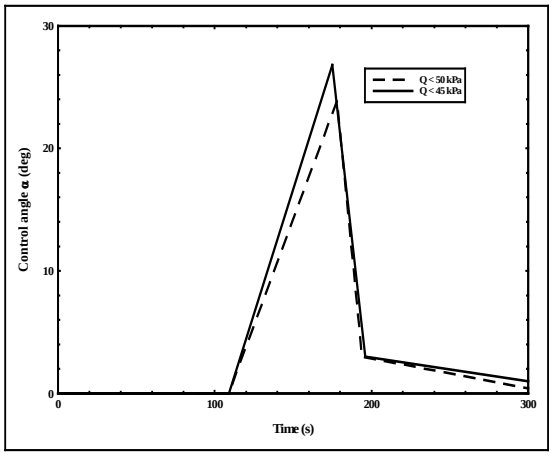

Fig.5 Control history for Q constraint only

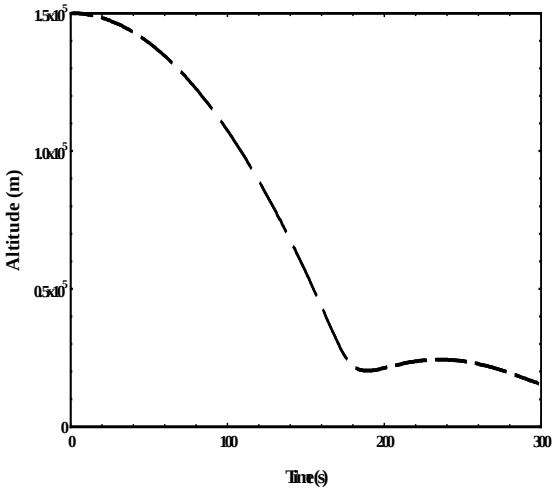

Fig.6 Altitude history for both Q & q constraint

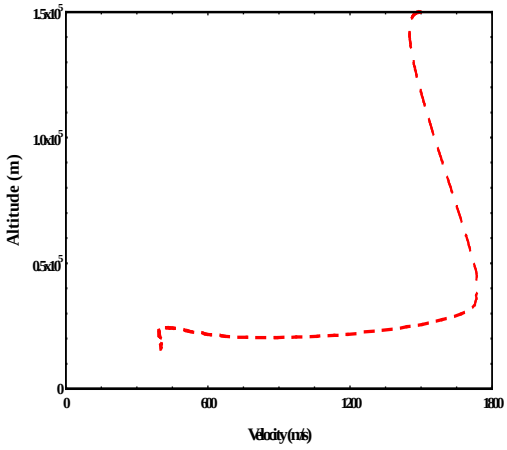

Fig.7 Velocity history for both Q & q constraint

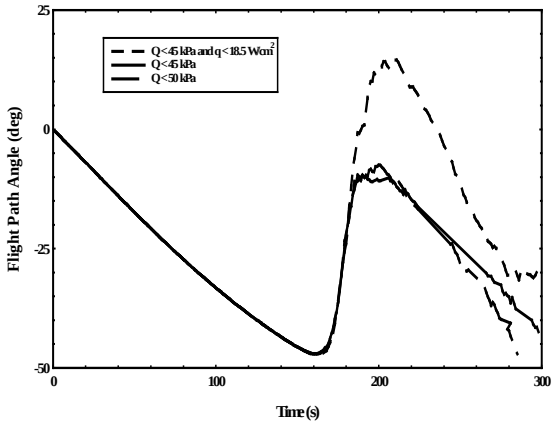

Fig.8 Flight path angle history

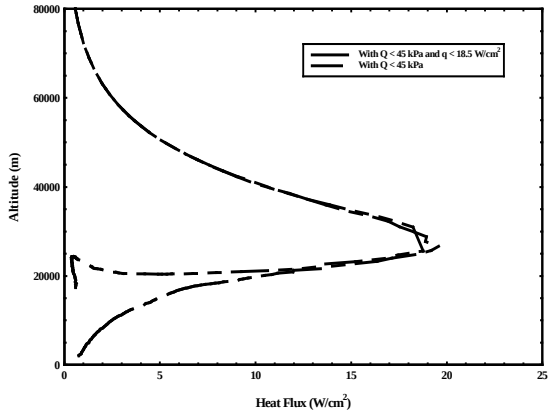

Fig.9 Heat flux history history

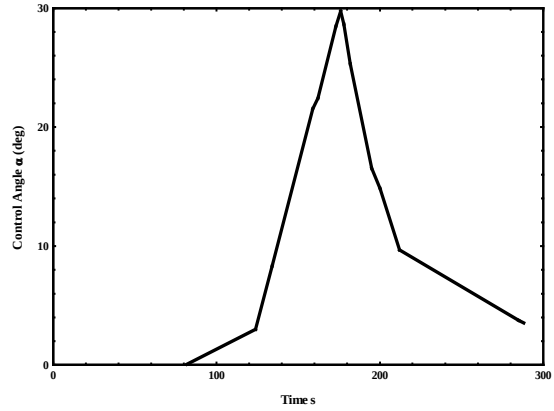

Fig.10 Control history for both Q & q constraint

For the reference trajectory, control U can be given as a function of time (Similar to Space Shuttle[8])as shown in Fig.11. The trend matches well with the reference trajectory of Space Shuttle[8]. The drag acceleration profile as given in Fig.12 also matches well (in trend) with that of space shuttle.

Summary

Reentry trajectory optimization problem is solved using genetic algorithm for a typical RLV-Technology Demonstrator. Dynamic pressure and heat flux are used as constraints. The results are comparable to that of Space Shuttle.

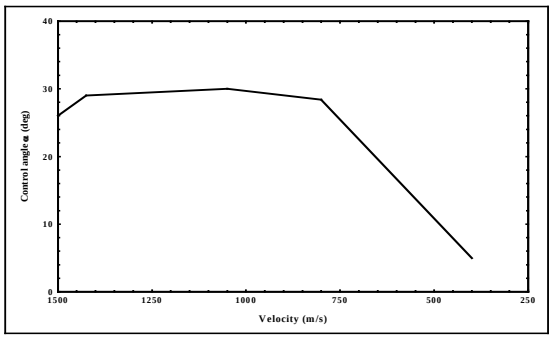

Fig.11 Reference Control

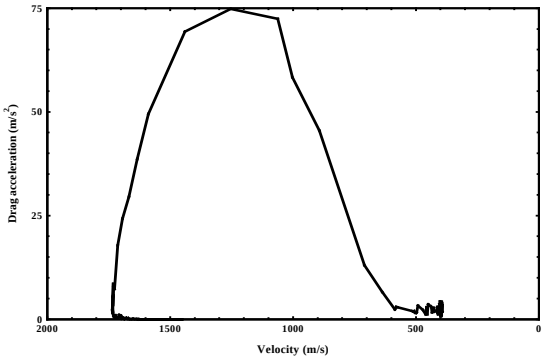

Fig.12 Drag acceleration

Acknowledgement

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